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MIDLAKES:
A Coordinated Hydrologic Response Model for the Middle Great Lakes

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MIDLAKES: A COORDINATED HYDROLOGIC RESPONSE MODEL FOR THE MIDDLE GREAT LAKES

Anne H. Clites and Deborah H. Lee

Abstract. A new model for simulating quarter-monthly lake levels and connecting channel flows for the middle Great Lakes (Lakes Michigan, Huron, St. Clair, and Erie) has been developed under the auspices of the Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data. The new middle lakes model, MIDLAKES, is part of a larger project to develop a common American and Canadian Great Lakes regulation and routing model for both operational and research purposes. MIDLAKES is the first step in the development of the coordinated model and is expected to evolve as it is incorporated with Lakes Superior and Ontario regulation and routing modules. MIDLAKES incorporates several improvements over existing models. It utilizes a state-of-the-art finite difference solution, is programmed in modules, and is fully documented. The new model is independent of datum, units, and connecting channel stage-discharge relationships. The result is a versatile model which will be useful for operational regulation and forecasting, evaluation of alternative lake regulation plans, simulation of historical conditions, and assessment of impacts due to channel changes, diversions, and climate change. The increased availability and improved documentation of the new model may also make it useful as an educational tool. MIDLAKES was verified by comparing its computed 1900-1989 monthly mean levels and outflows to those of the Basis of Comparison prepared for the International Joint Commission's Levels Reference Study. The annual mean difference between MIDLAKES and BOC values was -0.2 cm for levels for each lake and -3 to -4 m^3/s for outflows. Model results were also compared to recorded levels and flows for the period 1974-1989. The annual mean difference between modeled and recorded values ranged from 0.6 to 1.9 cm for levels and -7 to 4 m^3/s for outflows. The model was also tested using transposed climate and $2xCO_2$ climate scenarios and was found to be numerically robust for extreme water supply conditions. The model was evaluated for mass conservation. Mass loss or gain over a 90-year simulation period, as a percentage of mean lake outflows, was 0.4%, 5.7%, and 0.7% for Lakes Michigan-Huron, Lake St. Clair and Lake Erie, respectively.

1.0 INTRODUCTION

When Great Lakes water levels and outflows are significantly above or below their long-term averages, attention is drawn to the subject of prediction and regulation of Great Lakes levels. The NOAA Great Lakes Environmental Research Laboratory, the U.S. Army Corps of Engineers, and Environment Canada all maintain numerical models used to compute Great Lakes levels and outflows. These models are used for regulation of Lakes Superior and Ontario, and for forecasting levels and connecting channel flows. They are also used to assess the impacts of climate change, diversions, and alternative regulation plans.

Despite significant efforts to ensure that the models yield consistent results, small differences exist that confuses and reduces the credibility of these tools. In 1992, the Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data ("Coordinating Committee") discussed the need for a coordinated routing model that would be fully sanctioned by all relevant agencies from both countries, eliminating the need for constantly comparing model results. This model would utilize state-of-the-art numerical solution techniques, be fully documented internally and externally, and would be made flexible and robust enough to handle the most extreme climate change or regulation scenarios. An ad hoc committee made up of representatives from all three agencies was established in 1993 (Appendix I). A work plan and programming standards were established to develop a coordinated hydrologic response model in three phases: the middle lakes routing model, the Lake Superior regulation model, and the Lake Ontario regulation model. This technical memorandum documents the development of MIDLAKES, the middle lakes routing model, and serves as a user's manual. The middle lakes routing portion of the coordinated model is expected to evolve as it is integrated with Lakes Superior and Ontario regulation and routing. Some differences may exist between the model documented here and the final model in its entirety.

MIDLAKES is a new hydrologic response model for the unregulated middle lakes: Lakes Michigan, Huron, St. Clair and Erie, including the Upper Niagara River. Connecting channel flows may be represented by either single or double stage discharge relationships. Initial conditions for a model run are beginning-of-quarter-month water levels for the three lakes. Lakes Michigan and Huron are treated as one lake by the model. Inputs to the model include Lake Superior outflows, net basin supplies or their supply components (over-lake precipitation, basin runoff, and lake evaporation), diversions, consumptive use, groundwater, and ice and weed retardation for Lakes Michigan-Huron, St. Clair, and Erie. All of these parameters are read in on a quarter-month basis. Other user-supplied information required to use MIDLAKES includes lake surface areas, number of increments per quarter-month (minimum of 6), start and end month and year, and stage-discharge relationship constants.

Program MIDLAKES offers several improvements over the models it succeeds. It is computationally efficient, utilizing a state-of-the-art finite difference solution. It is programmed in modules and completely documented. The model is independent of datum, units, and connecting channel stage-discharge relationships. Running MIDLAKES requires the alteration of only one input data file that allows the user to specify stage-discharge relationship parameters as well as input file names, time step size and start and end dates. Input and output data files are in a conventional ASCII format. MIDLAKES is easy to modify for specific research applications because it is well documented both internally and externally. It also offers the choice of either a double or single gauge stage-discharge equation for the Niagara River. Previous models have represented the Niagara with a single-gage equation. This enhancement allows more flexibility in use of the model to evaluate management options such as use of the Chippawa-Grass Island Pool in ameliorating adverse Lake Erie levels during periods of extreme water supplies.

2.0 MODEL DEVELOPMENT

MIDLAKES calculates levels and flows for the middle lakes based on the 'hydrologic' finite difference solution described by Quinn (1978). The implicit finite difference approach uses the following outflow equation, applied to all three lakes:

$$(1)\qquad QO_{j,m} = QO_{j,t} + 1/2\left(\frac{\delta QO_j}{\delta t}\right)dt$$

where *QO* is outflow, *m* indicates the mean of the timestep, *t* represents outflow at time t, and *j* = 0,1, 2, 3, or 4 for Lakes Superior, Michigan-Huron, St. Clair, Erie, and Upper Niagara River levels, respectively. The generalized version of the stage-discharge equation is:

$$(2)\qquad QO_{j,t} = K_j\left[\varphi_j Z_{j,t} + \left(1-\varphi_j\right)Z_{j+1,t} - ym_j\right]^{a_j}\left(Z_{j,t} + Z_{j+1,t}\right)^{b_j}$$

where Z is the mean lake surface elevation, K is the outflow equation coefficient (a function of mean channel cross-section area and roughness), ym is the mean channel bottom elevation, and a and b are depth and fall exponents, respectively. The φ term allows a user-specified weighting of lake levels ($0 \leq \varphi \leq 1$) which allows the user to give the upstream or downstream lake more emphasis in the outflow determination. The following continuity equation expressed in terms of change in lake storage is solved for each of the three lakes:

$$(3)\quad P_{j,m} + R_{j,m} + QO_{j-1,m} - QR_{j-1,m} \pm D_{j,m} \pm G_{j,m} = EV_{j,m} + CU_{j,m} + QO_{j,m} - QR_{j,m} + A_j\left(\frac{\Delta Z_j^{t\to t+\Delta t}}{\Delta t}\right)$$

Components of the model include quarter-monthly mean overlake precipitation (P), runoff (R), ice-weed retardation (QR), evaporation rate (EV), diversion (D), groundwater (G), and consumptive use (CU). The lake surface area, A, is assumed to be constant over time for each lake. Substituting equation (2) into equation (1) and then equation (1) into equation (3) yields three equations which can be solved simultaneously to find the change in water levels, ΔZ_1, ΔZ_2, and ΔZ_3 from time *t* to $t+\Delta t$. The detailed derivations for each lake are shown in Appendix II.

Since either a double or a single gage relationship can represent each of the connecting channels, there are eight possible solution sets for the entire system. Summing all the known parameters (precipitation, runoff, evaporation, consumptive use, diversions, groundwater) on the left-hand side and algebraically rearranging the right hand side as a summation of products of ΔZ_j yields three equations in three unknowns:

$$(4)\qquad C_1 = c_{1,1}\Delta Z_1 + c_{1,2}\Delta Z_2 + c_{1,3}\Delta Z_3 \qquad \textit{Michigan}-\textit{Huron}$$

$$(5)\qquad C_2 = c_{2,1}\Delta Z_1 + c_{2,2}\Delta Z_2 + c_{2,3}\Delta Z_3 \qquad \textit{St.Clair}$$

$$(6)\qquad C_3 = c_{3,1}\Delta Z_1 + c_{3,2}\Delta Z_2 + c_{3,3}\Delta Z_3 \qquad \textit{Erie}$$

Arranged in matrix form:

$$(7) \qquad \begin{vmatrix} c_{1,1} & c_{1,2} & c_{1,3} \\ c_{2,1} & c_{2,2} & c_{2,3} \\ c_{3,1} & c_{3,2} & c_{3,3} \end{vmatrix} \begin{vmatrix} \Delta Z_1 \\ \Delta Z_2 \\ \Delta Z_3 \end{vmatrix} = \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix}$$

Solving these equations simultaneously yields the unknowns ΔZ_1, ΔZ_2, and ΔZ_3. The new lake surface elevations are then given by:

$$(8) \qquad Z_{j,t+\Delta t} = Z_{j,t} + \Delta Z_j$$

3.0 MODEL OPERATION

3.1 Overview

Program MIDLAKES was written in Fortran 77 (Lahey Version 5.00) and effort was made to keep the code as machine-independent as possible for ease of portability. The ad hoc committee adopted programming standards to guide the development of this and its companion programs in the new coordinated routing model. According to these programming standards, MIDLAKES reads in data one year at a time, limiting array sizes. Calculations are performed on a quarter-monthly basis. Supplies may be read in as either net basin supplies or as hydrologic components. The model was designed to handle extreme supply conditions. Additional details concerning programming standards are found in Appendix III.

Subroutine **INITDATA** initializes the model run by reading the input file, **initdata.*ext***, where "*.ext*" implies a user-defined file extension. The simulation dates, file extension, method of inputting supplies, discharge equation parameters, number of time steps, initial lake levels and all needed input and output filenames are specified by **initdata.*ext***. This is the only input file that requires user-manipulation. The stage-discharge equation coefficients and exponents: K, ym, a, b, and φ (equation 2) are initialized by **initdata.*ext*** for each of the three connecting channels: St. Clair River, Detroit River, and Niagara River. Lake levels may be expressed in terms of meters or feet, referenced to the same datum. The stage-discharge relationship parameters, initial levels, and input files must use the same units and datum. The **initdata.*ext*** file is read only once for the duration of the run. Figure 1 illustrates the basic computational logic of MIDLAKES.

Supplies are read in one year at a time. Next, the quarter-monthly loop is entered where lake levels are computed incrementally. A minimum of six increments is recommended for numerical stability. Within this loop, subroutines **COEFF** and **MATRIX** are called. **COEFF** calculates the nine matrix coefficients for the combination of gages and discharge equation parameters indicated in subroutine **INITDATA** ($c_{1,1}$ through $c_{3,3}$ in equations 4, 5, 6, and 7). Several calls are made to Function **OUTFLOW** from **COEFF.** Function **OUTFLOW** performs the stage-discharge equation calculations. **COEFF** also calculates left hand terms (C_1, C_2, C_3) needed to perform the matrix solution. These terms are quarter-monthly summations of net basin supply, diversions, consumptive use, inflow, outflow, and ice/weed retardation, all

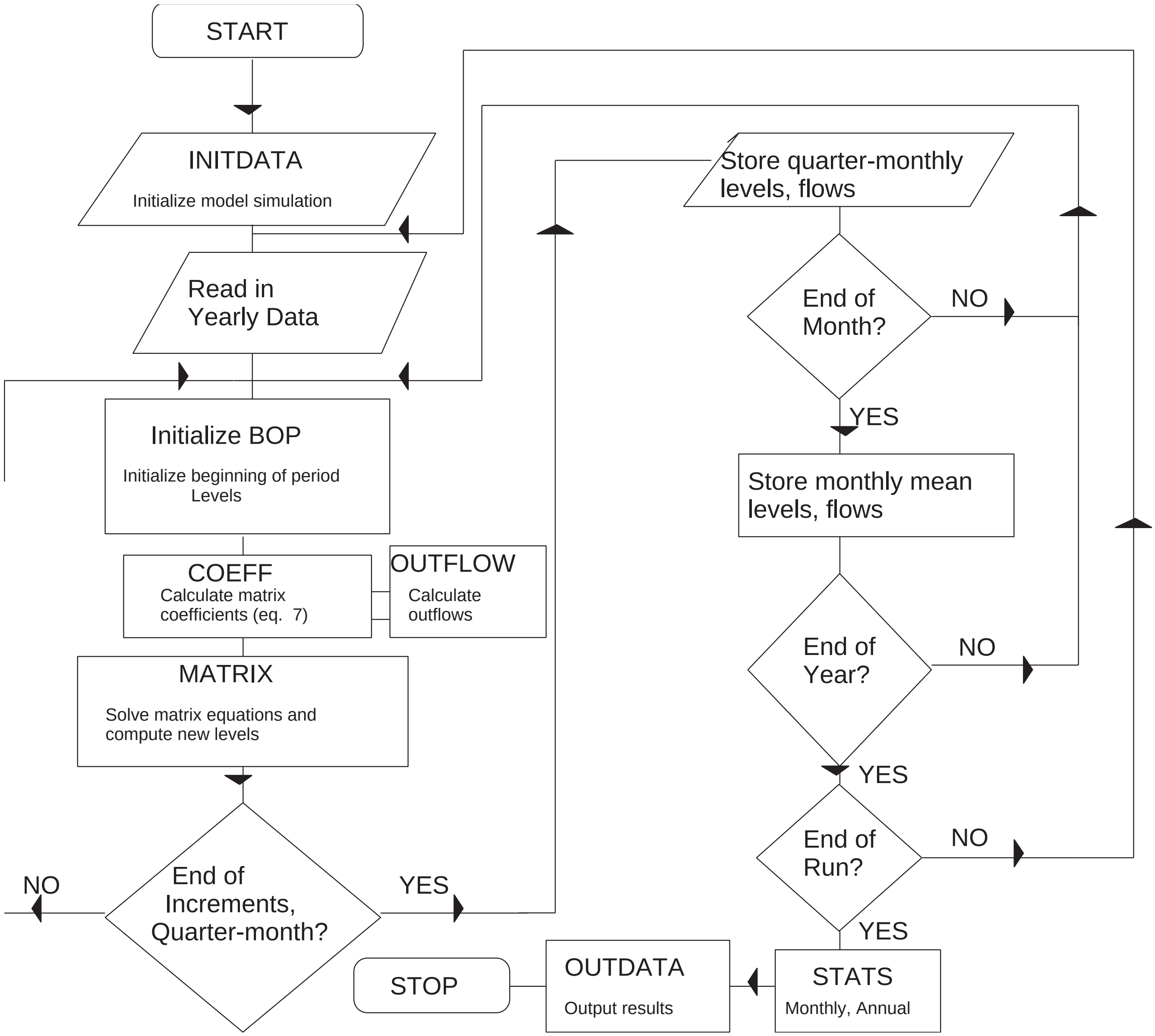


Figure 1. Schematic diagram of MIDLAKES.

known quantities. Immediately upon return from **COEFF** to the main, subroutine **MATRIX** is called. Subroutine **MATRIX** performs the matrix calculations using the nine coefficients determined by **COEFF**. **MATRIX** returns the values of the newly calculated lake levels to the main program. At the conclusion of the quarter-monthly and monthly loops, mean levels and flows are saved for statistics. The Fortran code for MIDLAKES is found in Appendix IV.

Subroutines **STATS** and **OUTDATA** are called at the end of the model run. **STATS** performs statistical calculations for all beginning-of-month levels, monthly mean levels, and monthly mean outflows. It calculates monthly and annual means, standard deviations, and monthly maximums and minimums. Subroutine **OUTDATA** creates separate output files for each lake and parameter (beginning-of-month level, mean level, mean flow).

3.2 Input/Output

The only input file designed for user manipulation is **initdata.*ext***. It allows the user several header lines, specifies number of runs and file extension to be used for output files, simulation dates, lake surface areas, and discharge equation parameters. The user should note that output files will bear the file extension specified inside the **initdata.*ext*** file, which may or may not be the same as the **initdata.*ext*** file extension. This file also provides access to the time step (number of increments per quarter-month), initial lake levels, and file names for all required input data files. A sample **initdata.boc** file is included in Appendix V.

Data files required for each lake include quarter-monthly values of net basin supplies or hydrologic components (over-lake precipitation, basin runoff, and over-lake evaporation), consumptive use, groundwater, diversions, ice/weed retardation, and Lake Superior outflows. Chippawa-Grass Island Pool end-of-quarter levels are required when a double gage relationship is used for the Niagara River. All input and output files are ASCII files. A conventional file format is used for input data: four lines per year; one line per quarter-month with the year and twelve data values in the format I4, 12F8.0 or 12F8.2 following three header lines. Input data file header lines include 1) lake name, 2) data type and possibly source, 3) units, years, format.

The file naming convention prescribed in the programming standards (Appendix III) follows the general rule: “lo-t-typ-op.ext” where “lo” is a two-digit location specifier (mh,sc,er); “t” is a one-digit time period indicator (m for monthly; q for quarter-monthly); “typ” is the data type (nbs for net basin supply, lev for water levels, etc.); “op” is a two-digit optional specifier; and “ext” is the user-defined file extension, usually specifying a particular run. For example, “mhqice.boc” contains the quarter-monthly Michigan-Huron flow retardation experienced by ice or weed build-up in the St. Clair River used for the Basis of Comparison (BOC) verification run.

Signage of the diversion, consumptive use, groundwater, and ice/weed retardation files is very important. Diversions out of the basin must be negative in the input files (Chicago Diversion; Welland Canal). Positive diversions are assumed to be into the basin. Consumptive uses and ice/weed retardation are always withdrawn in a water balance equation, so these inputs are assumed to be positive in sign. Groundwater should be signed in the same way as diversions: positive if the groundwater is a source;

negative if it is a sink.

Output from MIDLAKES simulations are beginning-of-month levels, monthly mean levels, and monthly mean outflows for Lakes Michigan-Huron, Lake St. Clair, and Lake Erie, and quarter-monthly Lake Ontario inflows. Output filenames follow the same convention as input filenames, for example, "**ermlev.boc**" is monthly mean Lake Erie levels from the BOC comparison simulation, and, similarly, "**mhblev.boc**" is beginning-of-month levels for Lakes Michigan-Huron. Although all input files are required to be in quarter-monthly format, most output files are monthly. The exception is the "**onqinflo.*ext***" output file, quarter-monthly Lake Ontario inflows saved in quarter-monthly format as needed for input to the Lake Ontario regulation plan model. "**Onminflo.*ext***" contains Niagara River outflows plus the Welland Canal Diversion. "**Ermflo.*ext***" is strictly monthly mean Niagara River outflows.

Two longer summary files are created by MIDLAKES. "**Summary.*ext***" includes initial conditions, monthly means, beginning-of-month levels, outflows, and net basin supplies for the three lakes for the duration of run as well as some monthly statistics for the period of run: mean, standard deviation, maximum and minimum levels. A sample "**summary.*ext***" file for the Basis of Comparison simulation is found in Appendix VI. "**Details.*ext***" is a larger summary file useful for debugging. It prints out all input and output values for each quarter-month. "**Details.*ext***" does not include statistics. The formats for the output files are similar to input formats with the exception that the output files print out month and quarter-month on each data line along with the year.

4.0 MODEL VERIFICATION

4.1 Replication of the Basis of Comparison

To verify MIDLAKES, the model replicated the Basis of Comparison (BOC) developed for the International Joint Commission's Levels Reference Study (Lee, 1993). The BOC is a 90-year series of monthly levels and flows representative of the present Great Lakes hydraulic conditions given the water supplies of the past (1900-1989). The same stage-discharge relationships, net basin supplies, diversions, ice/weed retardation values and Lake Superior outflows were input to MIDLAKES as were used for the BOC. The stage-discharge equations were derived in English units, referenced to the International Great Lakes Datum of 1955 (IGLD55), which is how they are presented here. The following discharge equations, representing present channel conditions, were used in both models:

$$(9) \qquad QO_{1,t} = 84.1168\left(\left(Z_{1,t} + Z_{2,t}\right)/2 - 543.4\right)^2 \left(Z_{1,t} - Z_{2,t}\right)^{0.5} \quad Michigan - Huron$$

$$(10) \qquad QO_{2,t} = 128.0849\left(Z_{2,t} - 543.81\right)^2 \left(Z_{2,t} - Z_{3,t}\right)^{0.5} \quad St.Clair$$

$$(11) \qquad QO_{3,t} = 260.5\left(Z_{3,t} - 550.11\right)^{2.2} \quad Erie$$

To summarize the difference between the two sets of levels and flows, four statistical comparisons were made: 1) the mean monthly difference, 2) the monthly standard deviation of differences; 3) the monthly maximum positive difference; and, 4) the monthly maximum negative difference. The results of these comparisons are reported in meters and cubic meters per second in Table 1.

The results from the MIDLAKES run compare very favorably with the BOC set of levels and outflows produced by the U.S. Army Corps of Engineers' Plan77A (Lake Superior regulation and middle lakes routing model) using the same input files for the years 1900-1989. MIDLAKES monthly mean levels are on average 0.2 centimeters lower than BOC monthly mean levels for each lake. Maximum and minimum differences are within acceptable tolerances.

Table 2 reveals an equally good comparison between the flows produced by MIDLAKES and the BOC flows. MIDLAKES mean monthly flows are slightly lower than BOC flows, generally by less than 10 m^3/s. The small differences in flows and levels can be attributed to the different numerical schemes employed. MIDLAKES uses the actual number of days in a month versus an average month length used in Plan 77A. MIDLAKES also does not use the extensive numerical rounding found in the Corps' model.

Table 1. MIDLAKES-BOC, Comparison of Monthly Values. Differences between monthly values in meters for the period 1900-1989.

	Lake Michigan-Huron				Lake St. Clair				Lake Erie			
	Mean	+Max	-Max	Std	Mean	+Max	-Max	Std	Mean	+Max	-Max	Std
Jan	-.003	.009	-.012	.005	-.002	.009	-.015	.005	-.002	.009	-.012	.004
Feb	-.002	.009	-.015	.005	-.002	.006	-.012	.004	-.003	.009	-.018	.005
Mar	-.002	.009	-.018	.005	0	.012	-.015	.006	-.002	.009	-.021	.006
Apr	-.002	.009	-.015	.005	-.001	.015	-.015	.005	-.001	.012	-.015	.005
May	-.002	.009	.015	.005	0	.012	-.015	.005	-.001	.009	-.015	.005
Jun	-.001	.012	-.015	.005	0	.009	-.015	.005	0	.009	-.015	.005
Jul	-.001	.009	-.015	.005	-.001	.012	-.015	.005	-.001	.009	-.015	.005
Aug	-.002	.009	-.015	.005	-.003	.006	-.012	.005	-.002	.009	-.015	.004
Sep	-.001	.009	-.012	.005	-.003	.006	-.015	.004	-.002	.009	-.015	.004
Oct	-.002	.009	-.012	.005	-.003	.006	-.015	.004	-.003	.006	-.015	.004
Nov	-.002	.009	-.012	.005	-.003	.003	-.015	.004	-.003	.003	-.015	.004
Dec	-.002	.009	-.012	.005	-.002	.009	-.012	.004	-.002	.006	-.012	.004
ANN	-.002	.009	-.015	.005	-.002	.006	-.012	.004	-.002	.006	-.015	.004

Table 2. MIDLAKES-BOC, Comparison of Monthly Outflows. Differences between monthly values in m^3/s for the period 1900-1989.

	St. Clair River				Detroit River				Niagara River + Welland Canal			
	Mean	+Diff	-Diff	Std	Mean	+Diff	-Diff	Std	Mean	+Diff	-Diff	Std
Jan	-14	1	-31	7	-14	1	-33	8	-3	18	-24	8
Feb	-12	4	-30	7	-9	6	-25	8	-5	18	-33	9
Mar	-6	9	-20	7	5	30	-23	10	-4	21	-42	12
Apr	-5	7	-25	7	4	24	-13	8	-1	30	-34	11
May	-3	9	-21	7	1	18	-21	8	-1	24	-34	10
Jun	-2	12	-20	8	0	19	-21	8	0	23	-33	10
Jul	-2	11	-19	7	-3	24	-19	8	-1	20	-32	9
Aug	0	15	-16	7	-5	8	-22	8	-4	19	-28	9
Sep	1	16	-14	7	-5	12	-22	8	-5	15	-26	8
Oct	1	14	-14	7	-6	10	-21	7	-5	12	-26	7
Nov	0	13	-16	7	-4	17	-19	8	-6	10	-25	7
Dec	-4	9	-18	7	-3	14	-18	8	-4	13	-22	7
ANN	-4	8	-19	6	-3	9	-18	6	-3	14	-29	8

4.2 Conservation of Mass

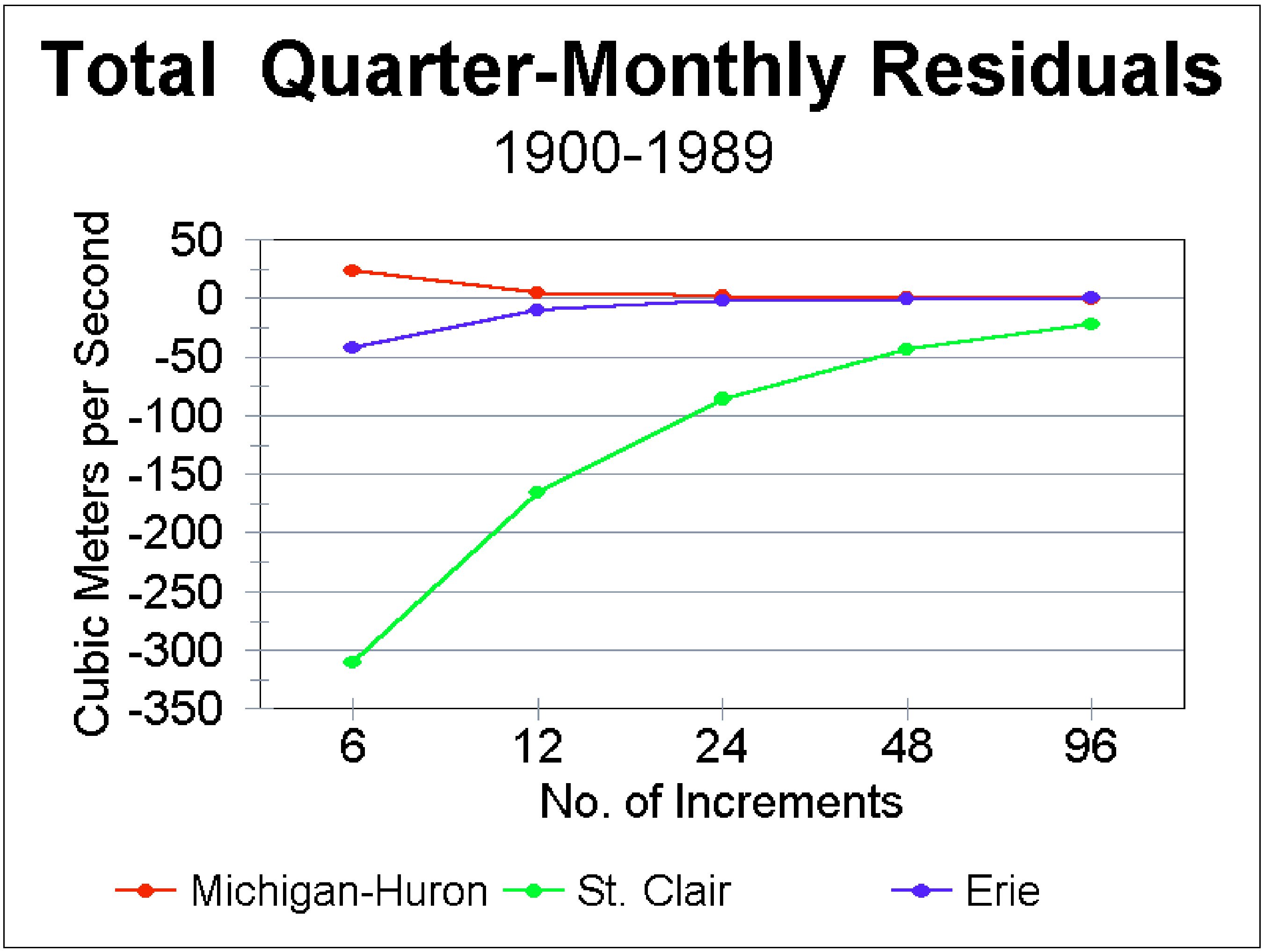
MIDLAKES was tested to ensure the model conserves mass. For each lake, quarter-monthly residuals were calculated by comparing change in storage to water balance summations. The total residual based on summing 90 years of quarter-monthly comparisons was then divided by lake surface area to yield a depth of residual over the lake. The over-lake residual is reported in Table 3 for six and twelve increments. As expected, the quarter-monthly residual decreases as the number of incremental calculations increases. The total residual is also expressed in Table 3 as a percentage of each lake's average annual outflow. The higher residuals for Lake St. Clair are a function of the very small lake surface area.

Sensitivity analysis was conducted to measure the effect of the increment size on the quarter-monthly residuals. The number of increments was increased from 6 to 96. Figure 2 summarizes the sensitivity analysis results. For numerical stability of the solution technique and for most purposes, the minimum number of increments is 6. The difference in results between using 6 increments and 12 increments is not significant to the nearest centimeter (hundredths of feet in English units). If results are being measured in millimeters, or there is a particular interest in Lake St. Clair, the user may want to increase the increments to 12 or more.

Table 3. 90-year Residuals.

Lake	6 increments		12 increments
	(cm)	(%)	(cm)
Lake Michigan-Huron	0.013	0.44	0.003
Lake St. Clair	18.349	5.74	9.754
Lake Erie	0.109	.074	0.026

Figure 2. Effect of increment size on quarter-monthly residuals.



4.3 Comparison to Recorded Lake Levels

4.3.1 Model Inputs

The ability of MIDLAKES to simulate recorded levels and outflows was tested for the period 1974-1989. This period was chosen because the hydraulic conditions of the middle lakes have been unchanged since 1973.

The same stage-discharge relationships (equations 9, 10, and 11) used for the BOC were used by MIDLAKES for this comparison. Net basin supplies were also used for the BOC comparison. Recorded monthly mean values for the Chicago diversion and Welland Canal (Croley and Hunter, 1994) were used in place of the monthly averages used previously. Recorded monthly Lake Superior outflows (Croley and Hunter, 1994) were used here in place of BOC Lake Superior outflows.

Since stage-discharge relationships cannot replicate recorded flows exactly, monthly values of weed and ice retardation for each of the three connecting channels were developed so that the differences between modeled and recorded flows only reflect model performance as nearly as possible. These monthly values were used here in place of the median values used for the BOC. The retardation values were calculated by subtracting monthly coordinated St. Clair and Detroit Rivers flows and reported Niagara River flows from flows calculated using the stage discharge equations and lake levels at the appropriate water level gages. Harbor Beach, St. Clair Shores, and Grosse Pointe gages were used in the Michigan-Huron stage-discharge equation (equation 9), St. Clair Shores, Grosse Pointe, and Cleveland gages were used in the Lake St. Clair equation (equation 10), and the Buffalo gage was used in the Lake Erie equation (equation 11). The difference between monthly mean recorded and calculated outflows represents the ice or weed retardation for that month. Negative values were set equal to zero because they are physically implausible.

A comparison of the monthly median values used for the BOC ice/weed retardation (Lee, 1993) and median values for the new calculated ice/weed retardation are shown in Table 4. Values differ due to different time periods for which the median values were calculated. The BOC values for Lakes Michigan-Huron and St. Clair were calculated using outflows from 1962-1989 and the new calculated values were based on outflows from 1974-1989. The BOC values for Lake Erie were calculated based on the same time period used here, 1974-1989.

Table 4. Calculated Ice and Weed Retardation Values.

St. Clair River Monthly Median Ice/Weed Retardation, m^3/s												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
BOC	538	453	113	0	0	0	0	0	0	0	0	85
MIDLAKES	298	273	45	0	0	0	0	0	0	0	0	70

Detroit River Monthly Median Ice/Weed Retardation, m^3/s												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
BOC	453	340	85	0	0	0	0	0	0	0	0	85
MIDLAKES	891	614	78	0	0	0	2	64	76	81	32	145

Niagara River Monthly Median Ice - Monthly Average Weed Retardation, m^3/s												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
BOC	226	170	113	142	0	57	226	142	85	57	0	142
MIDLAKES	126	63	20	57	16	62	225	147	88	56	42	63

4.3.2 Comparison to Recorded Levels: Results

The monthly mean levels and flows computed by MIDLAKES are compared to recorded monthly mean levels and recorded connecting channel outflows as reported by Croley and Hunter (1994). Modeled lake levels are compared to levels recorded at Harbor Beach, MI (Lake Michigan-Huron), St. Clair Shores, MI (Lake St. Clair), and Buffalo, NY (Lake Erie). The comparison is summarized in Tables 5 and 6. The monthly mean differences, maximum positive differences between monthly values, maximum negative differences between monthly values and difference in standard deviation are reported. The difference between MIDLAKES and recorded levels on an annual basis is 1.9 cm for Lakes Michigan-Huron, 1.4 cm for Lake St. Clair, and 0.6 cm for Lake Erie. Absolute maximum differences (positive or negative) are 6.1 cm, 14 cm, and 19.2 cm for Lakes Michigan-Huron, St. Clair, and Erie, respectively. These large maximum differences occur during fall and winter months when storm activity or ice retardation makes it difficult to accurately model lake levels. The differences between modeled and recorded outflows are small: in terms of annual means, -7 m^3/s for the St. Clair River, 1 m^3/s for the Detroit River, and 4 m^3/s for the Niagara River. Absolute maximum differences are 236 m^3/s, 434 m^3/s, and 440 m^3/s for Lakes Michigan-Huron, St. Clair, and Erie, respectively. As is the case for levels, these large discrepancies between monthly outflows take place during fall and winter months.

Table 5. Comparison of MIDLAKES and Recorded Levels. Differences between monthly statistics in meters for the period 1974-1989.

	Lake Michigan-Huron				Lake St. Clair				Lake Erie (Buffalo)			
	Mean	+Diff	-Diff	Std	Mean	+Diff	-Diff	Std	Mean	+Diff	-Diff	Std
Jan	.014	.052	-.012	.017	.039	.140	.003	.031	-.022	.076	-.110	.052
Feb	.020	.055	-.012	.017	.032	.073	-.003	.023	.054	.110	-.012	.038
Mar	.031	.061	0	.014	.026	.058	-.003	.019	.062	.110	.021	.024
Apr	.017	.043	-.027	.017	.02	.070	-.018	.025	.042	.067	.015	.015
May	.027	.046	.003	.012	.012	.040	-.015	.017	.040	.073	.015	.015
Jun	.027	.043	-.003	.012	.011	.058	-.037	.024	.023	.058	.012	.021
Jul	.014	.034	-.012	.013	-.003	.030	-.046	.023	.007	.030	.015	.017
Aug	.014	.034	-.018	.018	-.009	.037	-.043	.021	.006	.064	-.034	.023
Sep	.016	.046	-.024	.020	-.006	.021	-.064	.022	.004	.049	-.027	.021
Oct	.016	.049	-.009	.016	.002	.034	-.049	.020	-.020	.037	-.107	.038
Nov	.021	.052	-.003	.018	.014	.046	-.012	.019	-.055	.037	-.183	.047
Dec	.016	.034	-.027	.017	.033	.101	-.012	.028	-.070	.034	-.192	.058
ANN	.019	.045	-.012	.023	.014	.059	-.025	.023	.006	.062	-.053	.031

Table 6. Comparison of MIDLAKES and Recorded Outflows. Differences between monthly statistics in m^3/s for the period 1974-1989.

	St. Clair River				Detroit River				Niagara River			
	Mean	+Diff	-Diff	Std	Mean	+Diff	-Diff	Std	Mean	+Diff	-Diff	Std
Jan	-23	46	-236	67	11	434	-100	120	-48	161	-235	109
Feb	2	106	-79	46	43	195	-100	76	96	243	-31	85
Mar	26	83	-59	38	28	140	-74	59	126	215	14	55
Apr	-26	51	-123	47	-21	58	-136	55	72	135	-59	46
May	-22	45	-120	50	-11	52	-114	49	54	115	-44	41
Jun	-17	37	-109	44	-10	59	-95	45	47	132	-30	53
Jul	-23	34	-96	31	-20	39	-79	31	15	70	-33	35
Aug	-11	41	-67	36	-19	47	-84	38	14	139	-68	51
Sep	5	89	-110	54	-4	106	-108	51	7	104	-57	45
Oct	7	56	-48	30	-5	73	-56	39	-47	78	-211	76
Nov	10	80	-63	39	-4	35	-58	33	-128	45	-368	95
Dec	-13	58	-115	45	28	246	-90	84	-161	20	-440	118
ANN	-7	61	-102	44	1	124	-91	57	4	121	-130	67

Figures 3, 4, and 5 compare the monthly mean computed levels to the recorded levels for Lakes Michigan-Huron, St. Clair, and Erie, respectively. As shown in these figures, MIDLAKES replicates seasonal and annual trends. Figure 6, a plot of the differences between modeled Lake Erie levels and the levels recorded at Buffalo, NY, reveals a strong seasonal signal. This strong seasonal signal is also apparent in Figure 7, a plot of the differences between recorded monthly mean levels at Buffalo, NY and Cleveland, OH. This signal is a result of wind set-up, which results in higher lake levels at the eastern (Buffalo) end of the lake. The wind effect is more pronounced on Lake Erie than other lakes because its primary axis is directly aligned with the prevailing wind. The effect is strongest during the months of November, December, and January. Figure 7 shows that the monthly mean Lake Erie level at Buffalo in the fall and winter is often more than 10 centimeters and sometimes over 20 cm higher than the level at Cleveland. The average effect of the wind on the lake level between Cleveland and Buffalo based on the period 1888-1958 was found by the Coordinating Committee (1976) to be 7.0 centimeters for November, 9.1 cm for December and 10.4 cm for January.

During the fall and early winter of nearly every year, the MIDLAKES model seems to severely underestimate the Lake Erie level at Buffalo due in large part to the wind set-up, as illustrated by Figure 5. The same seasonal effect is evident in the difference between modeled and recorded outflows (Figure 8). The MIDLAKES model more closely simulates Lake Erie levels measured at Cleveland as shown in Figure 9 despite the fact that the Lake Erie stage-fall discharge equation was calibrated using levels at Buffalo. This comparison illustrates a limitation of the MIDLAKES routing model that users should keep in mind. MIDLAKES cannot reproduce the recorded levels and flows of Lake Erie to the same degree of accuracy as is possible for Lakes Michigan-Huron and Lake St. Clair because Lake Erie outflows and levels are strongly affected by seasonal wind patterns.

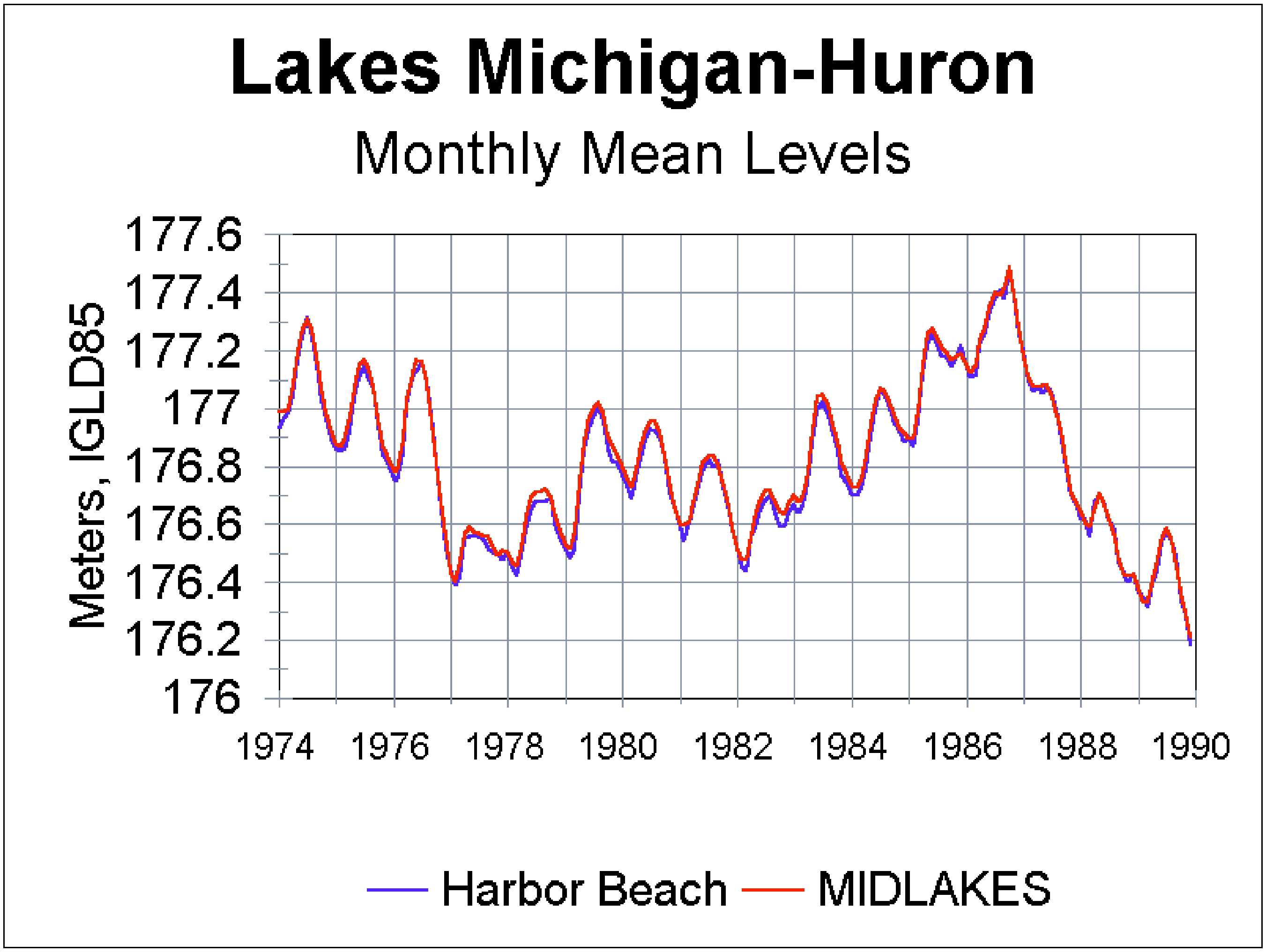


Figure 3. Comparison of 1974-1989 modeled monthly Lake Michigan-Huron levels with levels recorded at Harbor Beach, MI.

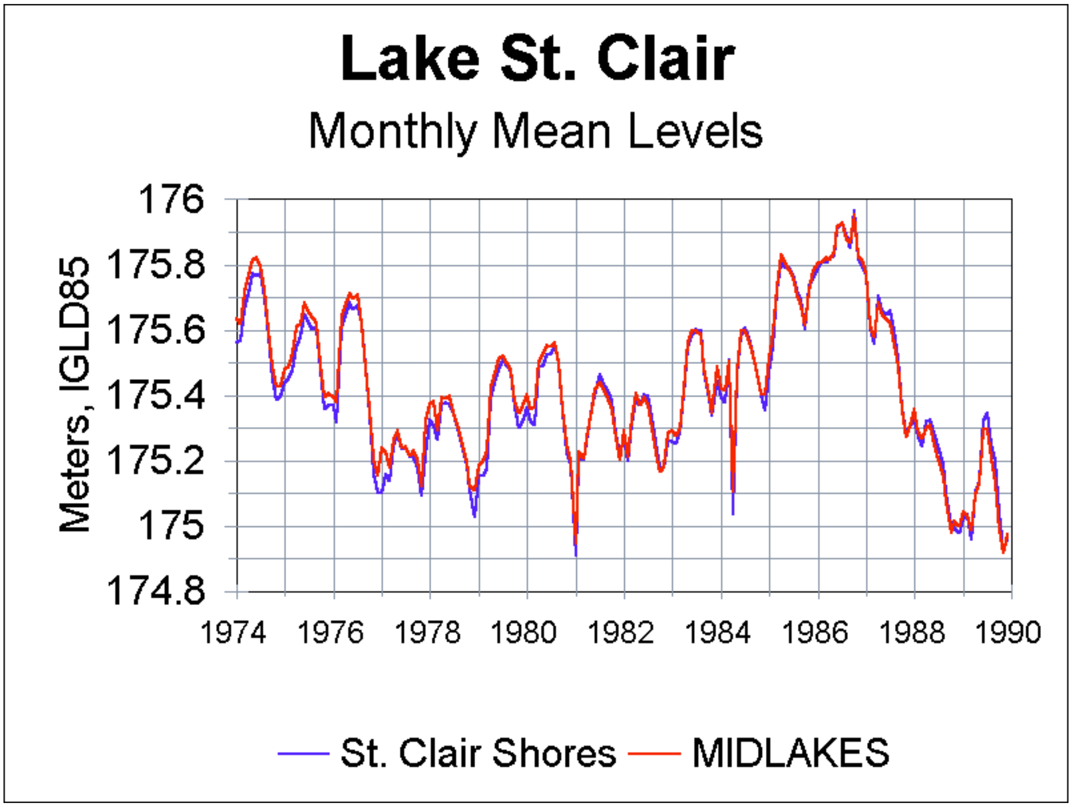


Figure 4. Comparison of 1974-1989 modeled monthly Lake St. Clair levels with levels recorded at St. Clair Shores, MI.

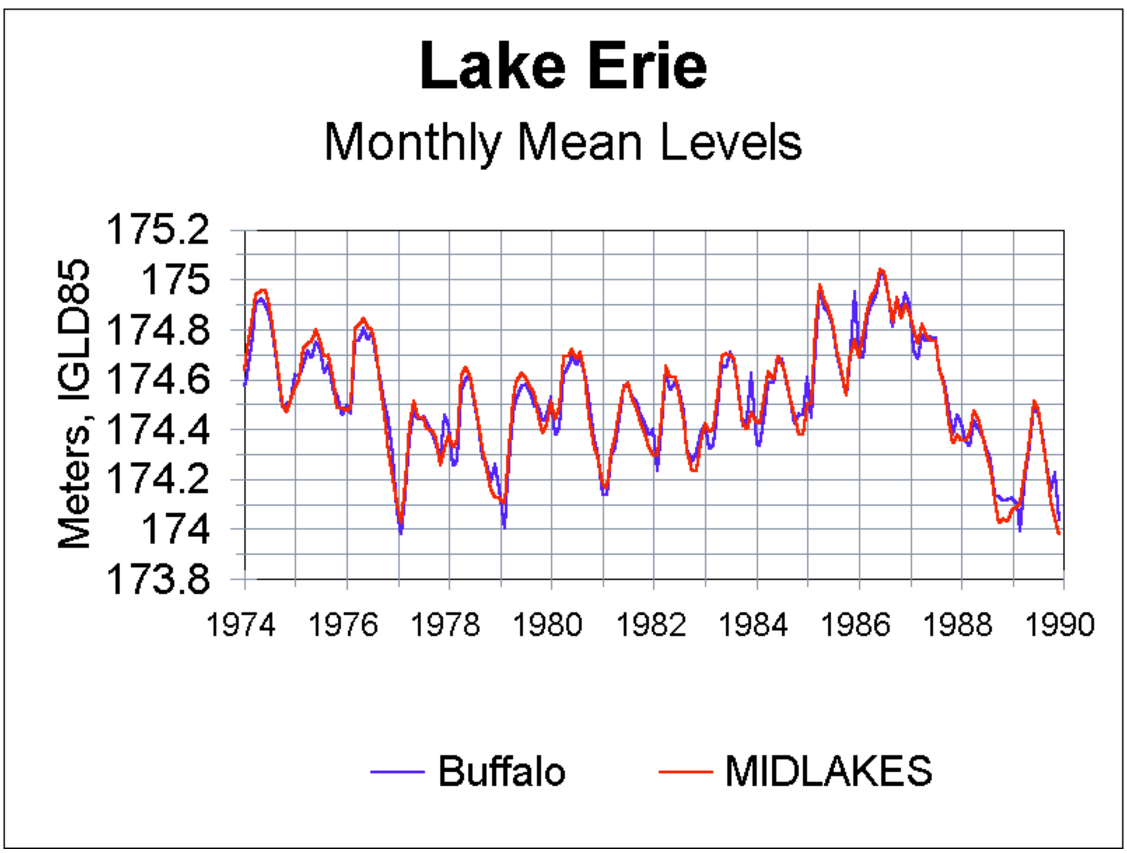


Figure 5. Comparison of 1974-1989 modeled monthly Lake Erie levels and levels recorded at Buffalo, N.Y.

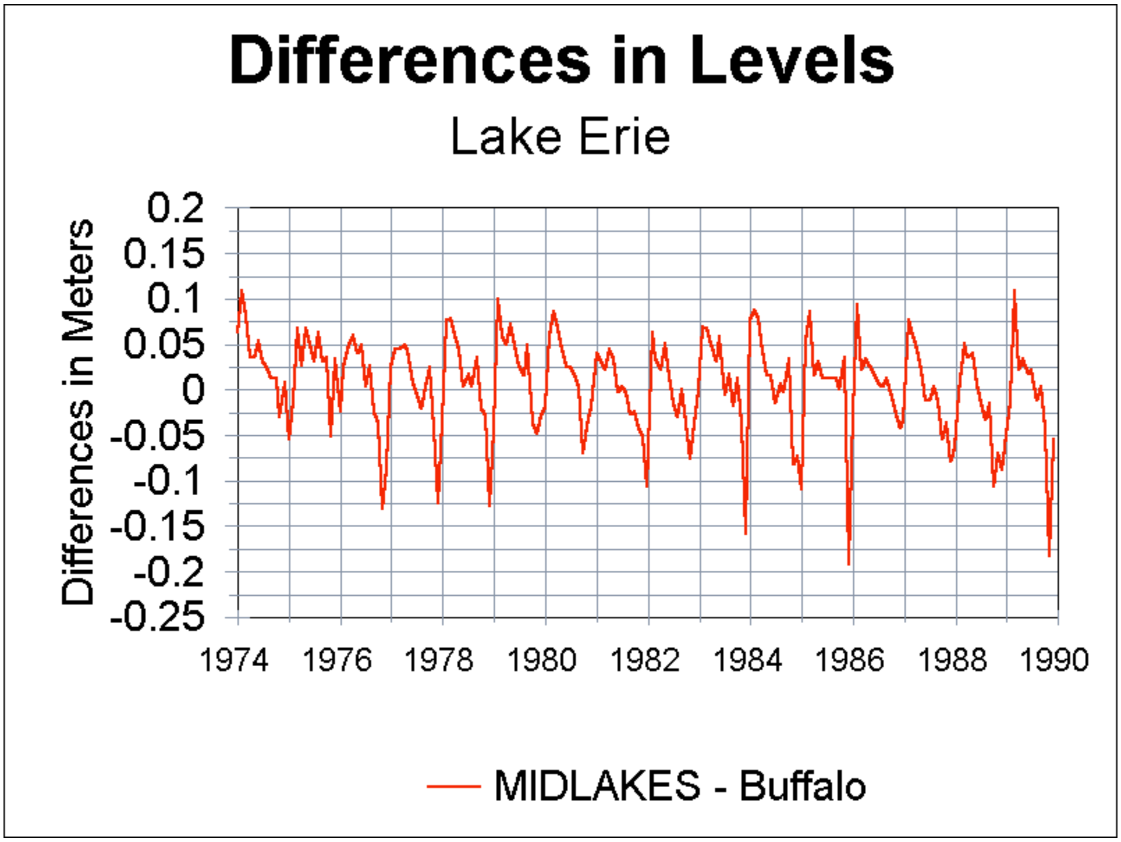


Figure 6. Differences between 1974-1989 modeled montly Lake Erie levels and levels recorded at Buffalo, NY.

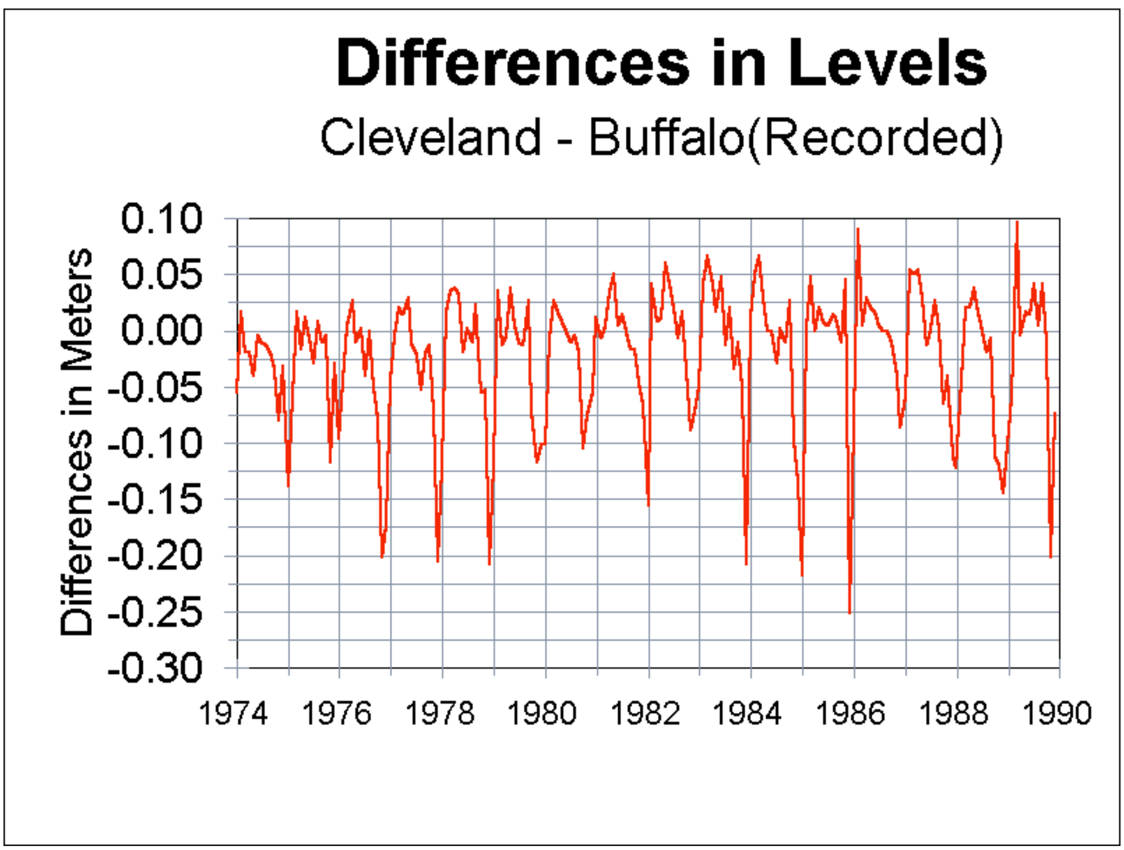


Figure 7. Differences between 1974-1989 levels recorded at Buffalo, NY and Cleveland, OH.

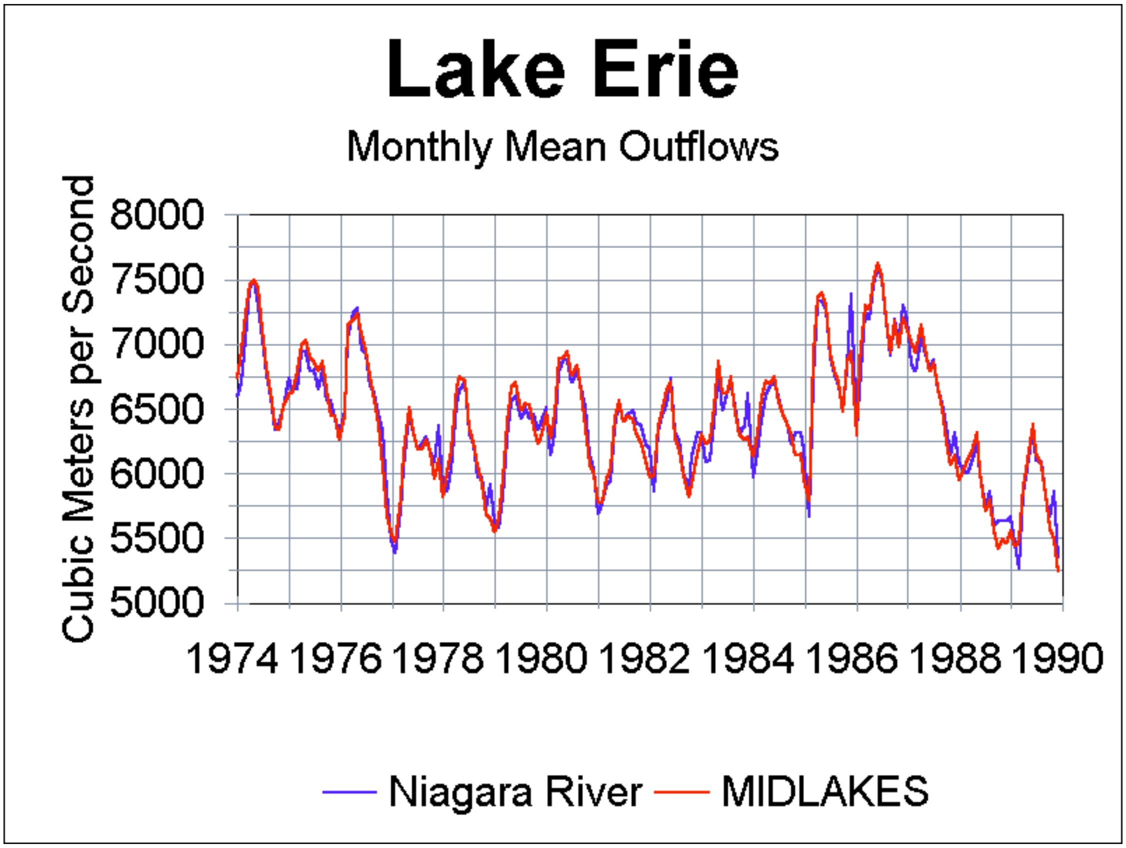


Figure 8. Differences between 1974-1989 modeled monthly outlows for Lake Erie and recorded Niagara River outflows.

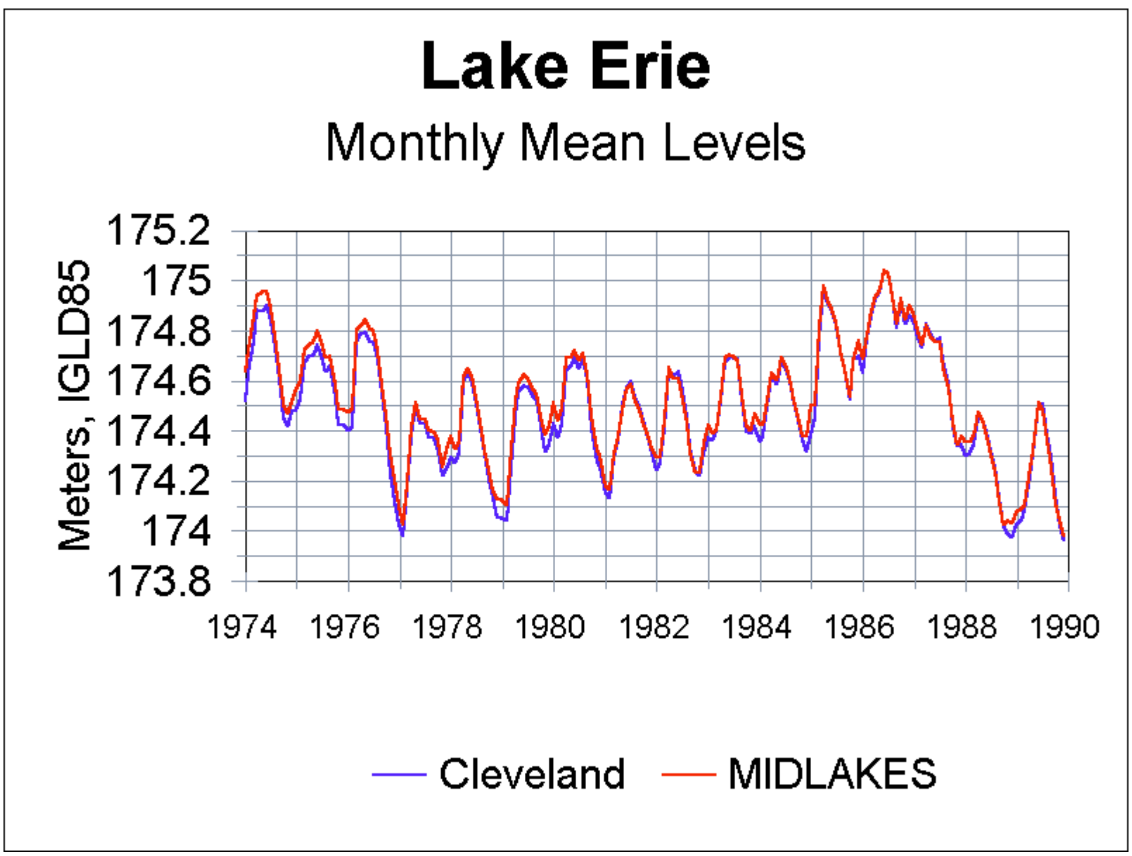


Figure 9. Comparison of 1974-1989 modeled Lake Erie levels to recorded levels at Cleveland, OH.

4.4 Tests of Model Robustness

4.4.1 Stage-Discharge Equation Combinations

With three connecting channels, each with the option of using a stage-discharge equation based on one or two gages, there are a total of eight possible gage configurations MIDLAKES must be able to handle. In trying to replicate the results from the "BOC" and in trying to simulate recorded levels and outflows, case 3 was tested (double gage relationships for Michigan-Huron and St. Clair; singe-gage relationship for Erie). Case 4 has also been tested in a study of the effects of managing the level of the Chippawa-Grass Island Pool on Lake Erie levels (Lee, et. al, 1998). The other 6 cases were tested here to ensure that the MIDLAKES model can handle all possible gage combinations. The same input files were used for these model runs as for the "BOC" run for the period 1900-1989. To carry out these tests, single-gage relationships had to be derived for the St. Clair and Detroit Rivers. This was accomplished very simply, since these equations were only to be used for testing. August BOC values for monthly mean level and flow for 1964 (low) and 1986 (high) were used to perform a linear regression which yielded the following single-gage relationships:

(12) $QO_{1,t} = 255.26(Z_{1,t} - 550.98)^2$ *Michigan-Huron*

(13) $QO_{2,t} = 338.14(Z_{2,t} - 549.87)^2$ *St. Clair*

The double gage equation for Lake Erie outflow was derived based on recorded monthly mean Niagara River flows and monthly mean water levels recorded at Material Dock (Chippawa-Grass Island Pool gage) and Buffalo gages for the ice and weed-free months of May, November, and December, 1981-1987. The equation is based theoretically on Manning's Equation for steady flow and fit to the data using regression analysis. It represents the present hydraulic regime of the river:

(14) $QO_{3,t} = 491.93\left(0.6Z_{3,t} + 0.4Z_{4,t} - 548.08\right)^{5/3}\left(Z_{3,t} - Z_{4,t}\right)^{0.5}$ *Erie*

where $Z_{3,t}$ is the monthly mean water level at the Buffalo gage and $Z_{4,t}$ is the monthly mean level at the Material Dock gage.

An 18-gate control structure was built partially spanning the Niagara River above the falls to allow increased diversion of water for hydropower generation. This structure, completed in 1963, changed the level and management regime of the Chippawa-Grass Island Pool (CGIP). The International Niagara Board of Control oversees management of the CGIP levels to ensure that river flows are in compliance with the Boundary Waters Treaty of 1909. A Board of Control directive in 1973 set the long-term mean level of the CGIP at 561 feet, IGLD55. One of the enhancements that the MIDLAKES model offers is the option to represent the Niagara River with a double gage relationship. Impacts of management directives for the CGIP can now be assessed.

Shown in Table 7 are the annual mean lake levels resulted from running MIDLAKES using all possible combinations of equations 9 through 14.

Table 7. Annual Mean Lake Levels Resulting from MIDLAKES Runs. All possible gage combinations tested.

Case No.	# of gages (Mich-Hur, St. Clair, Erie)	Michigan-Huron	St. Clair	Erie
1	(1,1,1)	578.28	573.80	570.85
2	(2,1,1)	578.30	573.80	570.85
3	(2,2,1)	578.25	573.72	570.85
4	(2,2,2)	578.21	573.65	570.76
5	(1,2,1)	578.40	573.71	570.85
6	(1,2,2)	578.27	573.65	570.76
7	(1,1,2)	578.28	573.80	570.76
8	(2,1,2)	578.30	573.80	570.76

4.4.2 Extreme Climate Scenarios

One of the goals of this improved routing model is to be able to handle extreme high or low supplies. To test MIDLAKES' handling of extremely dry conditions, net basin supplies developed during a Great Lakes climate change study by the Midwest Climate Center and GLERL (Croley et. al, 1995) were used to run the model. In the 1995 study, four different regional climates were superimposed on the Great Lakes basin to see how lake levels and flows would be affected. A modified version of Plan77A was used to simulate levels and flows. Net basin supplies were calculated at GLERL based on meteorological data provided by the Midwest Climate Center. The resulting scenarios were:

BASE: Normal Great Lakes climate.

MCC1: Move the Great Lakes region 6 degrees south and 10 degrees west.

MCC2: Move the Great Lakes region 6 degrees south.

MCC3: Move the Great Lakes region 10 degrees south and 11 degrees west.

MCC4: Move the Great Lakes region 10 degrees south and 5 degrees west.

Scenarios MCC1 and MCC3 represent very dry conditions as compared to the Base, resulting in negative annual net basin supplies for the Superior and Michigan-Huron basins. Scenarios MCC2 and MCC4 also represent drier than normal conditions, but not as extreme as MCC1 and MCC3.

To test MIDLAKES' ability to handle extremely dry conditions, the model was run using the net basin supplies developed for these five scenarios. Aside from net basin supplies and Superior inflows, all other inputs (diversions, ice/weed retardation, etc.) were the same as those used in the Basis of Comparison. These model runs were for the years 1951-1990. MIDLAKES and Plan77A are not directly comparable; MIDLAKES does not model Lake Superior regulation. For the purposes of these comparisons, Plan77A Superior outflows were used as an input to MIDLAKES. MIDLAKES was run to steady state for each case as Plan77A was for these climate studies. The resulting mean annual levels and flows were compared to those produced by the Plan77A runs to see how well MIDLAKES handles extremely low water supplies. The comparisons are shown in Tables 8 and 9 for the Michigan-Huron basin; results for St. Clair and Erie basins were comparable.

Table 8. MIDLAKES vs. Plan77A (in parentheses) Levels for Lake Michigan-Huron.

	Base	MCC1	MCC2	MCC3	MCC4
Level, m	176.65 (176.65)	173.31 (173.32)	176.41 (176.42)	173.14 (173.16)	176.40 (176.42)
Std. Deviation	0.38 (0.38)	0.57 (0.57)	0.37 (0.36)	0.44 (0.43)	0.57 (0.56)
Maximum	177.35 (177.36)	174.47 (174.47)	177.17 (177.17)	173.90 (173.92)	177.47 (177.48)
Minimum	175.82 (175.83)	172.29 (172.31)	175.81 (175.82)	172.23 (172.27)	175.44 (175.47)
Range	1.53 (1.53)	2.18 (2.16)	1.36 (1.35)	1.67 (1.65)	2.03 (2.01)

For a test of high supplies, the net basin supplies developed to simulate the summer of 1993 Mississippi flood conditions (Quinn et al., 1997) were used as input to MIDLAKES. Superior outflows from Plan77A were used as input to MIDLAKES. All diversion and ice/weed retardation files were from the Basis of Comparison as was the case with the Plan77A run used for comparison. The model runs simulate lake levels and outflows that may have happened from May through October, 1993 if the water supply conditions experienced by the upper Mississippi River basin during that period had occurred in the Great Lakes basin.

Table 9. MIDLAKES vs. Plan77A (in parentheses) Outflows for Lake Michigan-Huron.

	Base	MCC1	MCC2	MCC3	MCC4
Outflows, m^3/s	5816 (5818)	1992 (1997)	5307 (5319)	1857 (1872)	5366 (5388)
Std. Deviation	512 (511)	465 (462)	510 (503)	313 (313)	759 (756)
Maximum	6837 (6837)	3160 (3167)	6320 (6319)	2508 (2518)	6723 (6735)
Minimum	4736 (4740)	1109 (1122)	4531 (4548)	1259 (1278)	4158 (4197)
Range	2101 (2097)	2051 (2045)	1789 (1771)	1249 (1240)	2565 (2538)

The results for the two different models are similar. The mean difference between any one month simulated by MIDLAKES and the same month simulated by Plan77A was 4 centimeters (range: 0 to 9 centimeters). The mean difference between monthly outflows was 7 m^3/s (range: -8 to 21 m^3/s). Graphs of the comparison of levels and outflows follow in Figure 10.

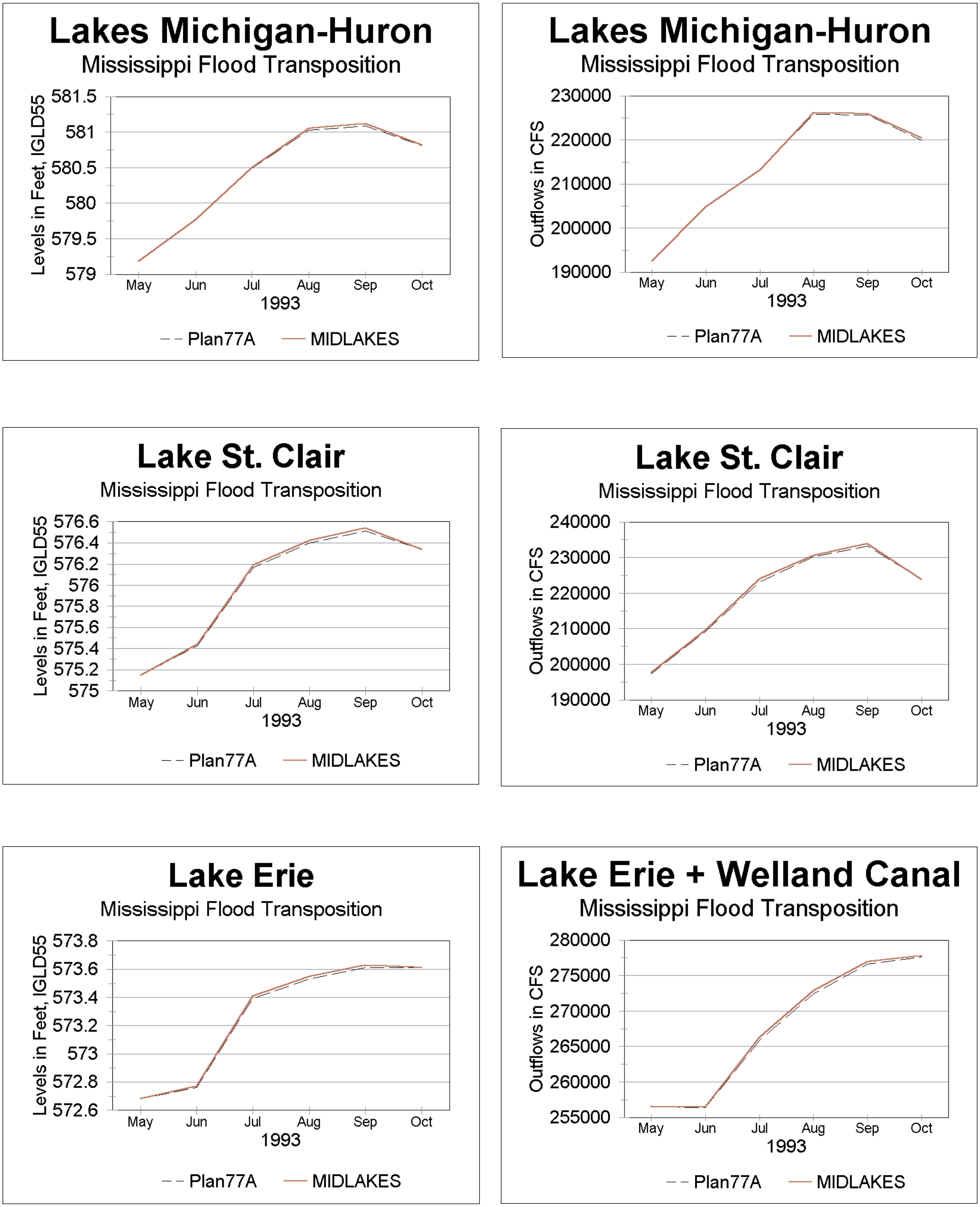


Figure 10. Mississippi flood transposition: MIDLAKES vs. Plan77A.

5.0 SUMMARY

A new model has been developed for simulating quarter-monthly levels and connecting channel flows for the middle Great Lakes (Michigan-Huron through Erie). MIDLAKES was developed at the Great Lakes Environmental Research Laboratory under the auspices of the multi-agency Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data. MIDLAKES will continue to evolve as it is incorporated into the Coordinated Great Lakes Regulation and Routing Model that will include the regulation plans for Lake Superior and Lake Ontario.

MIDLAKES replicates the Basis of Comparison set of levels and flows developed for the 1993 IJC Levels Reference study. The annual mean difference between MIDLAKES and BOC values for the period 1900-1989 was -0.2 cm for levels and -3 to -4 m^3/s for outflows for all lakes. The comparison of MIDLAKES to recorded levels and outflows was made for the period 1974-1989, a period representing the present hydraulic regimen of the middle lakes. The maximum differences between monthly values ranged from -19.2 to 14 cm for levels and -440 to 434 m^3/s for outflows. The extremes in those ranges are primarily from fall and winter months when wind set-up causes hard-to-model fluctuations in levels and flows. The difference between monthly means for most months is less than 5 cm for levels and less than 50 m^3/s for flows. MIDLAKES was also tested and performed well for extreme climate scenarios. The model was evaluated for mass conservation. Mass loss or gain as a percentage of mean lake outflows ranged from 0.4 - 5.7%.

MIDLAKES is easier to use, better documented, and is a more flexible research tool than the middle-lakes routing models it replaces. The model is independent of datums, units, and connecting channel stage-discharge relationships. It offers the user the option of using a double gage relationship for Lake Erie outflows that did not exist in previous models. The improved finite-difference solution employed by MIDLAKES performs calculations on increments (6 minimum) of a quarter-month basis for levels and outflows. It is more computationally efficient than the method employed by older models. Monthly values are saved for final output and statistics, but quarter-monthly values are available in a detailed output file as well.

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APPENDIX I. Ad Hoc Committee Members and Associates

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APPENDIX II. Derivation of Continuity Equations

Subscripts:

0	Lake Superior
1	Lakes Michigan-Huron
2	Lake St. Clair
3	Lake Erie
4	Chippawa Grass Island Pool
t	denotes beginning of timestep
m	denotes mean of timestep

Variables:

P	Precipitation on the lake surface
R	Runoff from the lake's basin
EV	Evaporation from the lake surface
QO	Lake outflow
QR	Ice retardation
D	Diversion (+ into the lake, - out of the lake)
CU	Consumptive use
G	Groundwater contribution (+ into the lake, - out of the lake)
A	Lake surface area
Z	Lake or CGIP elevation
Δt	model timestep
φ	weighting coefficient (between 0 and 1) for stage-discharge relationship
K	empirical coefficient for stage-discharge relationship
a	empirical coefficient for stage-discharge relationship
b	empirical coefficient for stage-discharge relationship
ym	empirical bottowm elevation for stage-discharge relationship
δ	partial derivative
ΔZ	change in lake surface elevation for Δt

Lake Erie

Given:

(1) $QO_{3,m} = QO_{3,t} + 1/2\left(\frac{\delta QO_3}{\delta t}\right)dt$

(2) $QO_{3,t} = K_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3}(Z_3 - Z_4)^{b_3}$

(3) $\frac{\delta QO_3}{\delta t} = K_3 a_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3-1}[\varphi_3\frac{\Delta Z_3}{\Delta t} + (1-\varphi_3)\frac{\Delta Z_4}{\Delta t}](Z_3 - Z_4)^{b_3} + K_3 b_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3}(Z_3 - Z_4)^{b_3-1}(\frac{\Delta Z_3}{\Delta t} - \frac{\Delta Z_4}{\Delta t})$

Substituting (3) into (1):

(4) $QO_{3,m} = QO_{3,t} + 1/2\{K_3 a_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3-1}[\varphi_3\Delta Z_3 + (1-\varphi_3)\Delta Z_4](Z_3 - Z_4)^{b_3} + K_3 b_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3}(Z_3 - Z_4)^{b_3-1}(\Delta Z_3 - \Delta Z_4)\}$

Given:

(5) $P_3 + R_3 + QO_{2,m} - QR_2 \pm D_3 \pm G_3 = EV_3 + CU_3 + QO_{3,m} - QR_3 + A_3(\frac{\Delta Z_3}{\Delta t})$

Substituting (4) into (5):

$$P_3 + R_3 + QO_{2,m} - QR_2 \pm D_3 \pm G_3 = EV_3 + CU_3 + QO_{3,t} \tag{6}$$

$$+1/2\{K_3 a_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3-1}[\varphi_3 \Delta Z_3 + (1-\varphi_3)\Delta Z_4](Z_3 - Z_4)^{b_3} + K_3 b_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3}(Z_3 - Z_4)^{b_3-1}(\Delta Z_3 - \Delta Z_4)\} - QR_{3+} + A_3(\frac{\Delta Z_3}{\Delta t})$$

Substituting (2) into (6):

$$P_3 + R_3 + QO_{2,m} - QR_2 \pm D_3 \pm G_3 = EV_3 + CU_3 + K_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3)^{a_3}(Z_3 - Z_4)^{b_3} - QR_3 \tag{7}$$

$$+1/2\{K_3 a_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3-1}[\varphi_3 \Delta Z_3 + (1-\varphi_3)\Delta Z_4](Z_3 - Z_4)^{b_3} + K_3 b_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3}(Z_3 - Z_4)^{b_3-1}(\Delta Z_3 - \Delta Z_4)\} + A_3(\frac{\Delta Z_3}{\Delta t})$$

Substituting (10-12) for $QO_{2,m}$:

$$P_3 + R_3 + K_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2}(Z_2 - Z_3)^{b_2} - QR_2 \tag{8}$$

$$+1/2\{K_2 a_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2-1}[\varphi_2 \Delta Z_2 + (1-\varphi_2)\Delta Z_3](Z_2 - Z_3)^{b_2} + K_2 b_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2}(Z_2 - Z_3)^{b_2-1}(\Delta Z_2 - \Delta Z_3)\} \pm D_3 \pm G_3$$

$$= EV_3 + CU_3 + K_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3)^{a_3}(Z_3 - Z_4)^{b_3} - QR_3$$

$$+1/2\{K_3 a_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3-1}[\varphi_3 \Delta Z_3 + (1-\varphi_3)\Delta Z_4](Z_3 - Z_4)^{b_3} + K_3 b_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3}(Z_3 - Z_4)^{b_3-1}(\Delta Z_3 - \Delta Z_4)\} + A_3(\frac{\Delta Z_3}{\Delta t})$$

Rearranging:

$$
\begin{aligned}
(9)\quad & P_3 + R_3 - EV_3 \pm D_3 \pm G_3 - CU_3 + K_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2}(Z_2 - Z_3)^{b_2} - QR_2 - K_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3)^{a_3}(Z_3 - Z_4)^{b_3} + QR_3 \\
& +\{1/2K_3 b_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3}(Z_3 - Z_4)^{b_3-1} - 1/2K_3 a_3(1-\varphi_3)[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3-1}(Z_3 - Z_4)^{b_3}\}\Delta Z_4 \\
& = (0)\Delta Z_1 \\
& -\{1/2K_2 b_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2}(Z_2 - Z_3)^{b_2-1} + 1/2K_2 a_2\varphi_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2-1}(Z_2 - Z_3)^{b_2}\}\Delta Z_2 \\
& +\{\frac{A_3}{\Delta t} + 1/2K_3 b_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3}(Z_3 - Z_4)^{b_3-1} + 1/2K_3 a_3\varphi_3[\varphi_3 Z_3 + (1-\varphi_3)Z_4 - ym_3]^{a_3-1}(Z_3 - Z_4)^{b_3} \\
& +1/2K_2 b_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2}(Z_2 - Z_3)^{b_2-1} - 1/2K_2 a_2(1-\varphi_2)[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2-1}(Z_2 - Z_3)^{b_2}\}\Delta Z_3
\end{aligned}
$$

Lake St. Clair:

Given:

(10) $$QO_{2,m} = QO_{2,t} + 1/2\left(\frac{\delta QO_2}{\delta t}\right)dt$$

(11) $$QO_{2,t} = K_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2}(Z_2 - Z_3)^{b_2}$$

(12) $$\frac{\delta QO_2}{\delta t} = K_2 a_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2-1}[\varphi_2\frac{\Delta Z_2}{\Delta t} + (1-\varphi_2)\frac{\Delta Z_3}{\Delta t}](Z_2 - Z_3)^{b_2} + K_2 b_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2}(Z_2 - Z_3)^{b_2-1}\left(\frac{\Delta Z_2}{\Delta t} - \frac{\Delta Z_3}{\Delta t}\right)$$

Substituting (12) into (10):

(13)

$$QO_{2,m} = QO_{2,t} + 1/2\{K_2 a_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2-1}[\varphi_2\Delta Z_2 + (1-\varphi_2)\Delta Z_3](Z_2 - Z_3)^{b_2} + K_2 b_2[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2]^{a_2}(Z_2 - Z_3)^{b_2-1}(\Delta Z_2 - \Delta Z_3)\}$$

Given:

(14) $$P_2 + R_2 + QO_{1,m} - QR_1 \pm D_2 \pm G_2 = EV_2 + CU_2 + QO_{2,m} - QR_2 + A_2\left(\frac{\Delta Z_2}{\Delta t}\right)$$

Substituting (13) into (14):

$$(15) \qquad P_2 + R_2 + QO_{1,m} - QR_1 \pm D_2 \pm G_2 = EV_2 + CU_2 + QO_{2,t}$$
$$+1/2\left\{K_2 a_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2-1}\left[\varphi_2 \Delta Z_2 + (1-\varphi_2)\Delta Z_3\right](Z_2 - Z_3)^{b_2} + K_2 b_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2} Z_2 - Z_3)^{b_2-1}(\Delta Z_2 - \Delta Z_3)\right\}$$
$$-QR_2 + A_2\left(\frac{\Delta Z_2}{\Delta t}\right)$$

Substituting (11) into (15):

$$(16) \qquad P_2 + R_2 + QO_{1,m} - QR_1 \pm D_2 \pm G_2 = EV_2 + CU_2 + K_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2}(Z_2 - Z_3)^{b_2} - QR_2$$
$$+1/2\left\{K_2 a_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2-1}\left[\varphi_2 \Delta Z_2 + (1-\varphi_2)\Delta Z_3\right](Z_2 - Z_3)^{b_2} + K_2 b_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2}(Z_2 - Z_3)^{b_2-1}(\Delta Z_2 - \Delta Z_3)\right\} + A_2\left(\frac{\Delta Z_2}{\Delta t}\right)$$

Substituting (19-21) for $QO_{1,m}$:

$$(17) \qquad P_2 + R_2 + K_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1} - QR_1$$
$$+1/2\left\{K_1 a_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1-1}\left[\varphi_1 \Delta Z_1 + (1-\varphi_1)\Delta Z_2\right](Z_1 - Z_2)^{b_1} + K_1 b_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1-1}(\Delta Z_1 - \Delta Z_2)\right\} \pm D_2 \pm G_2$$
$$= EV_2 + CU_2 + K_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2}(Z_2 - Z_3)^{b_2} - QR_2$$
$$+1/2\left\{K_2 a_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2-1}\left[\varphi_2 \Delta Z_2 + (1-\varphi_2)\Delta Z_3\right](Z_2 - Z_3)^{b_2} + K_2 b_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2}(Z_2 - Z_3)^{b_2-1}(\Delta Z_2 - \Delta Z_3)\right\} + A_2\left(\frac{\Delta Z_2}{\Delta t}\right)$$

Rearranging:

$$
\begin{aligned}
(18)\quad & P_2 + R_2 - EV_2 \pm D_2 \pm G_2 - CU_2 + K_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1} - QR_1 - K_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2}(Z_2 - Z_3)^{b_2} + QR_2 \\
& = -\left\{1/2K_1 b_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1 - 1} + 1/2K_1 a_1 \varphi_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1 - 1}(Z_1 - Z_2)^{b_1}\right\}\Delta Z_1 \\
& +\{\frac{A_2}{\Delta t} + 1/2K_2 b_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2}(Z_2 - Z_3)^{b_2 - 1} + 1/2K_2 a_2 \varphi_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2 - 1}(Z_2 - Z_3)^{b_2} \\
& -1/2K_1 b_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1 - 1} - 1/2K_1 a_1(1-\varphi_1)\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1 - 1}(Z_1 - Z_2)^{b_1}\}\Delta Z_2 \\
& -\{1/2K_2 b_2\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2}(Z_2 - Z_3)^{b_2 - 1} - 1/2K_2 a_2(1-\varphi_2)\left[\varphi_2 Z_2 + (1-\varphi_2)Z_3 - ym_2\right]^{a_2 - 1}(Z_2 - Z_3)^{b_2}\}\Delta Z_3
\end{aligned}
$$

Lakes Michigan-Huron:

Given:

(19) $$QO_{1,m} = QO_{1,t} + 1/2\left(\frac{\delta QO_1}{\delta t}\right)dt$$

(20) $$QO_{1,t} = K_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}\left(Z_1 - Z_2\right)^{b_1}$$

(21) $$\frac{\delta QO_1}{\delta t} = K_1 a_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1-1}\left[\varphi_1\frac{\Delta Z_1}{\Delta t} + (1-\varphi_1)\frac{\Delta Z_2}{\Delta t}\right](Z_1 - Z_2)^{b_1} + K_1 b_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}\left(Z_1 - Z_2\right)^{b_1-1}\left(\frac{\Delta Z_1}{\Delta t} - \frac{\Delta Z_2}{\Delta t}\right)$$

Substituting (21) into (19):

(22)

$$QO_{1,m} = QO_{1,t} + 1/2\left\{K_1 a_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1-1}\left[\varphi_1\Delta Z_1 + (1-\varphi_1)\Delta Z_2\right](Z_1 - Z_2)^{b_1} K_1 b_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}\left(Z_1 - Z_2\right)^{b_1-1}(\Delta Z_1 - \Delta Z_2)\right\}$$

Given:

(23) $$P_1 + R_1 + QO_{0,m} \pm D_1 \pm G_1 = EV_1 + CU_1 + QO_{1,m} - QR_1 + A_1\left(\frac{\Delta Z_1}{\Delta t}\right)$$

Substituting (22) into (23):

$$P_1 + R_1 + QO_{0,m} \pm D_1 \pm G_1 = EV_1 + CU_1 + QO_{1,t} \tag{24}$$

$$+1/2\left\{K_1 a_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1-1}\left[\varphi_1 \Delta Z_1 + (1-\varphi_1)\Delta Z_2\right](Z_1 - Z_2)^{b_1} + K_1 b_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1-1}(\Delta Z_1 - \Delta Z_2)\right\}$$

$$-QR_1 + A_1\left(\frac{\Delta Z_1}{\Delta t}\right)$$

Substituting (20) into (24):

$$P_1 + R_1 + QO_{0,m} \pm D_1 \pm G_1 = EV_1 + CU_1 + K_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1} - QR_1 \tag{25}$$

$$+1/2\left\{K_1 a_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1-1}\left[\varphi_1 \Delta Z_1 + (1-\varphi_1)\Delta Z_2\right](Z_1 - Z_2)^{b_1} + K_1 b_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1-1}(\Delta Z_1 - \Delta Z_2)\right\} + A_1\left(\frac{\Delta Z_1}{\Delta t}\right)$$

Rearranging ($QO_{0.m}$ is known):

$$P_1 + R_1 + QO_{0,m} \pm D_1 \pm G_1 - EV_1 - CU_1 - K_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1} + QR_1 \tag{26}$$

$$= \left\{\frac{A_1}{\Delta t} + 1/2K_1 a_1 \varphi_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1-1}(Z_1 - Z_2)^{b_1} + 1/2K_1 b_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1-1}\right\}\Delta Z_1$$

$$+\left\{1/2K_1 a_1(1-\varphi_1)\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1-1}(Z_1 - Z_2)^{b_1} - 1/2K_1 b_1\left[\varphi_1 Z_1 + (1-\varphi_1)Z_2 - ym_1\right]^{a_1}(Z_1 - Z_2)^{b_1-1}\right\}\Delta Z_2$$

$$+(0)\Delta Z_3$$

The determinants of the following matrices must be found in order to find the change in water level:

(27) $$detC = \begin{vmatrix} c_{1,1} & c_{1,2} & c_{1,3} \\ c_{2,1} & c_{2,2} & c_{2,3} \\ c_{3,1} & c_{3,2} & c_{3,3} \end{vmatrix}$$

(28) $$detZ_1 = \begin{vmatrix} C_1 & c_{1,2} & c_{1,3} \\ C_2 & c_{2,2} & c_{2,3} \\ C_3 & c_{3,2} & c_{3,3} \end{vmatrix}$$

(29) $$detZ_2 = \begin{vmatrix} c_{1,1} & C_1 & c_{1,3} \\ c_{2,1} & C_2 & c_{2,3} \\ c_{3,1} & C_3 & c_{3,3} \end{vmatrix}$$

(30) $$detZ_3 = \begin{vmatrix} c_{1,1} & c_{1,2} & C_1 \\ c_{2,1} & c_{2,2} & C_2 \\ c_{3,1} & c_{3,2} & C_3 \end{vmatrix}$$

After these matrices are solved, the end-of-increment change in water levels are determined:

(31) $$\Delta Z_1 = \frac{\det Z_1}{\det C}$$

(32) $$\Delta Z_2 = \frac{\det Z_2}{\det C}$$

(33) $$\Delta Z_3 = \frac{\det Z_3}{\det C}$$

APPENDIX III. Programming Standards

The general requirements set forth by the Ad Hoc subcommittee of the Coordinating Committee for the middle lakes routing module include:

generic Fortran
supplies input as NBS or components
extreme supply conditions permissible
input file names and initial conditions in one input file
elimination of most data statements
complete internal documentation
data read in one year at a time to limit array allocation

Datafile structure specifications:

- all ascii files
- missing data code: "-9999"
- top three lines of each data file reserved for documentation:
  - data descriptor
  - units
  - time period, Fortran format statement
- for monthly data, each line will contain the year followed by 12 monthly values
- for quartermonthly data, each record will contain the year, the quarter, and the 12 quarter-monthly data values
- each input file will only contain one type of data

Datafile naming convention:

- names will not exceed 12 characters, including the "." preceding the extension
- Characters 1 and 2 will be location designation:
  - sp - Lake Superior
  - mh - Lakes Michigan-Huron
  - sc - Lake St. Clair
  - er - Lake Erie
  - on - Lake Ontario
- Character 3 will indicate data period:
  - m - monthly
  - q - quartermonthly
  - b - beginning of month
  - d - daily
  - w - weekly
- Characters 4, 5, and 6 indicate data type:
  - nbs - net basin supply
  - ice - ice/weed retardation
  - lev - water levels
  - sar - surface area
  - evp - evaporation
  - flo - flow
  - div - diversion
  - run - runoff
  - pre - precipitation

con - consumptive use
gdw - groundwater

Characters 7 and 8 are optional characters preceding the 4 character extension, “.ext”, where “ext” is reserved for scenario identification.

Program structure:

The main program will call all modules.

Modules will be independent such that any can be altered without impacting the main program or other modules.

The main program will read in data which is universal to the computation of lake storage.

Each module will read in data and parameters specific to that module.

Computations will be in units of tens of cubic meters per second to 1 decimal place and in meters to 2 decimal places.

Documentation standards:

All variables passed to or from subroutines will be documented with a brief description, including data type (real or integer).

Each subroutine will have a descriptive header at the beginning.

Variable names are limited to 8 characters, lower case.

Subroutine names are limited to 8 characters, upper case.

Comment lines in proper English will be used as necessary to explain the logic.

APPENDIX IV. MIDLAKES Fortran Code
available via ftp:
ftp://ftp.glerl.noaa.gov/publications/tech_reports/glerl-109/midlakes.for

APPENDIX V. Sample MIDLAKES Input file -initdata.boc
available via ftp:
ftp://ftp.glerl.noaa.gov/publications/tech_reports/glerl-109/initdata.boc

Appendix VI. Sample MIDLAKES Output File - summary.boc
available via ftp:
ftp://ftp.glerl.noaa.gov/publications/tech_reports/glerl-109/summary.boc